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# Determining the Optimum Process Mean Based on Manufacturing Cost and Quality Loss

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**ABSTRAK**

Wen dan Mergen (1999) mengemukakan satu kaedah untuk menentukan min proses optimum berdasarkan pengimbangan kos untuk tidak menemukan had spesifikasi atas dan bawah. Walau bagaimanapun, mereka tidak mengambil kira kehilangan kualiti sesuatu produk yang ada di dalam spesifikasi model. Dalam kertas ini, dicadangkan model Wen dan Mergen (1999) yang telah diubah suai untuk menentukan min proses optimum mengandungi kos pembuatan dan kehilangan kualiti. Fungsi kehilangan kualiti kuadratik dan linear digunakan dalam mengukur kehilangan kualiti produk. Kos pembuatan dianggap berkadar negatif dan linear kepada lencongan daripada nilai sasaran apabila ciri kualiti adalah di dalam spesifikasi dan kos pembuatan malar berlaku apabila ciri kualiti melebihi spesifikasi.

**ABSTRACT**

Wen and Mergen (1999) presented a method for determining the optimum process mean based on balancing the cost of not meeting upper and lower specification limits. However, they have not considered the quality loss for a product which is within specifications in the model. In this paper, we propose the modified Wen and Mergen's (1999) model for determining the optimum process mean consisting of manufacturing cost and quality loss. The quadratic and linear quality loss functions are used in measuring the quality loss of product. The manufacturing cost is assumed to be negatively and linearly proportional to deviation from the target value when the quality characteristic is within specifications and constant manufacturing cost happens when the quality characteristic exceeds specifications.

**Keywords: Process control, process mean, quality loss function, specification limits, manufacturing cost**

## INTRODUCTION

Traditionally, the step quality loss function is used for deciding if the product is a conformance. If the quality characteristic of product is within specifications, then the product can be accepted and there is no quality loss. In 1986, Taguchi redefined that the product quality is the total losses to society, see Taguchi (1986). Hence, he proposed the quadratic quality loss function for reducing the expected losses. The advantage of the quadratic quality loss function is that we can evaluate losses in term of process mean and process standard deviation. However, the regular quadratic loss function is patently inappropriate in some situations. Trietsch (1999, p. 69) remarked that One such case occurs when the expected cost of exceeding the tolerance limits is not equal to the right and to the left of the target. Missing by cutting too much, for instance, may imply scrap,

while cutting too little only causes rework. When this is the case one possible response is fitting a loss function that is not symmetric and not necessarily quadratic. In some cases, a linear quality loss function is more appropriate in many industrial applications than that of quadratic quality loss function.

The optimum process mean setting has been a major topic of statistical process control. The selection of optimum process mean will directly affect the process defective rate, manufacturing cost, scrap cost, rework cost, and the cost of use. There are considerable attentions paid to the study of economic selection of process mean. The asymmetric quadratic quality loss function and asymmetric linear quality loss function are usually used in the related problems, e.g. Wu and Tang (1998), Li (1997, 1998, 2000, 2002a, 2002b), Li and Chirng (1999), Li and Cherng (2000), Maghsoodloo and Li (2000), Phillips and Cho (2000), Li and Chou (2001), and Li and Wu (2001,2002), etc.

Wen and Mergen (1999) presented a method for determining the optimum process mean based on balancing the cost of not meeting upper and lower specification limits. However, they have neglected to consider the quality loss for a product which is within specifications in the model. In this paper, we propose the modified Wen and Mergen's (1999) model for determining the optimum process mean consisting of manufacturing cost and quality loss. The quadratic and linear quality loss functions are used in measuring the quality loss of product. In order to reduce the expected quality losses of product, we have to reduce the bias (the deviation from the target value to process mean) and the standard deviation of the process. However, we usually face the manufacturing cost increases, e. g., inputting the high class of material, the advanced technology of machine and the high technology of people, when the quality improves. The manufacturing cost is assumed to be negatively and linearly proportional to deviation from the target value when the quality characteristic is within specifications and constant manufacturing cost happens when the quality characteristic exceeds specifications. The normal quality characteristic is considered in the modified model and a numerical example is provided for illustration.

## QUALITY LOSS FUNCTION

*Quadratic Quality Loss Function*

Assume that X represents the nominal-is-best type quality characteristic. For such quality characteristic, we have a nominal value or target value, and on each side of this value the performance of the product deteriorates as we move from the target value. For such characteristic, engineers generally have the lower and the upper specification limits. We can use the quality loss coefficients $k_1$ and $k_2$ to represent the left and right directions of deviation from the target value $T$. If tolerances for both sides are $D_L$ and $D_U$, the quality losses at the specification limits are defined as and , respectively. The asymmetric quadratic quality loss function is defined, see Taguchi (1986) as

$$L(X)=\begin{cases} D_L, & X<T_L \\ k_1(T-X)^2, & T_L \\ K_2(X-T)^2, & T<X\le T_U \\ D_U & X>T_U \end{cases} \qquad (1)$$

where

$T_U$ the upper specification limit
$T_L$ the lower specification limit

$$k_1 = \frac{D_L}{\Delta_1^2}$$

$$k2 = \frac{D_U}{\Delta_2^2}.$$

If $D_L = D_U$ and $\Delta_1 = \Delta_2$, Equation (1) is called symmetric quadratic quality loss function.

*Linear Quality Loss Function*

Under the same hypothesis as in subsection 2.1, the asymmetric linear quality loss function is defined, see Trietsch (1999) as

$$L(X)=\begin{cases} D_L, & X<T_L \\ k_1(T-X), & T_L \le X<T \\ k_2(X-T), & T<X\le T_U \\ D_U, & X>T_U \end{cases} \quad (2)$$

where

$$k_1 = \frac{D_L}{\Delta_1}$$

$$k_2 = \frac{D_U}{\Delta_2}.$$

Again if $D_L = D_U$ and $\Delta_1 = \Delta_2$, Equation (2) is called symmetric linear quality loss function.

## MANUFACTURING COST

For the nominal-is-best type product, the nominal value of quality characteristic is a constant. It is called design target. If the output value of quality characteristic is the design target, then the product has the highest evaluation. We usually need to adopt the advanced technology of manufacturing, machine, people and material to achieve the highest class output. That is, we usually face the manufacturing cost increases when the quality improves. Assume that the manufacturing cost is negatively and linearly proportional to deviation from the target value when the quality characteristic is within specifications and constant manufacturing cost happens when the quality characteristic exceeds specifications. Then we define the manufacturing cost function for the nominal-is-best type product as

$$M(X)=\begin{cases} a, & X<T_L \\ a+b\dfrac{X-T_L}{T-T_L}, & T_L \le X<T \\ a+b\dfrac{T_u-X}{T_U-T}, & T \le X \le T_U \\ a & X>T_U \end{cases} \tag{3}$$

where

$a$ the constant manufacturing cost

$b$ the variable manufacturing cost coefficient per unit.

## WEN AND MERGEN'S COST MODEL

There are three assumptions in Wen and Mergen's (1999) model: (1) the quality characteristic, say $X$, is normally distributed with unknown mean $\mu$ and known variance $\sigma^2$, or estimated in advance; (2) $X$ is nominal-is-best type; (3) the target value, say $T$, is the middle value of the specifications, i.e., $T = (T_U + T_L)/2$.

According to Wen and Mergen (1999, p. 508), if f represents the probability density function of $X$ and $\Phi$ is the distribution function of standard normal distribution, the total loss per item is

$$\begin{aligned} C_T &= D_U \int_{T_U}^{\infty} f(x)dx + D_L \int_{-\infty}^{T_L} f(x)dx \\ &= D_U\left[1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+D_L\Phi\left(\frac{T_L-\mu}{\sigma}\right) \end{aligned} \tag{4}$$

$C_T$ in Equation (4) is a function of $\mu$. In order to minimize $C_T$, Wen and Mergen (1999) consider the first and second derivative of Equation (4) with respect to $\mu$. Due to the fact that the second derivative is positive for all $\mu$, Wen and Mergen (1999) set the first derivative equal to zero and found the following optimal value of $\mu$, say $\mu^*$ that minimized the total loss per item:

$$\mu^* = T+\frac{\sigma^2}{T_U-T_L}\ln\frac{D_L}{D_U}. \tag{5}$$

## MODIFIED WEN AND MERGEN'S COST MODEL

*With Manufacturing Cost and Quadratic Quality Loss Function*

Assume that the normal quality characteristic $X$ has unknown mean $\mu$ and known variance $\sigma^2$, or estimated in advance. If $\Phi$ and $\phi$ represent the distribution function and the probability density function of standard normal distribution, the objective is to minimize the following total losses to society per item consisting of the producer's manufacturing cost and the customer's quadratic quality loss:

$$
\begin{aligned}
&C_{T_1} \\
&= D_U\left[1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+D_L\Phi\left(\frac{T_L-\mu}{\sigma}\right)+k_1\left\{\left[\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T-\mu}{\sigma}\right)-\sigma(T+\mu)\phi\left(\frac{T-\mu}{\sigma}\right)\right.\right. \\
&\quad -\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T_L-\mu}{\sigma}\right)+\sigma(T_L+\mu)\phi\left(\frac{T_L-\mu}{\sigma}\right)\Big]-2T\left[-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\right. \\
&\quad \left.\left.+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]+T^2\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]\right\} \\
&\quad +k_2\left\{\left[\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\sigma(T_U+\mu)\phi\left(\frac{T_U-\mu}{\sigma}\right)\right.\right. \\
&\quad -\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T-\mu}{\sigma}\right)+\sigma(T+\mu)\phi\left(\frac{T-\mu}{\sigma}\right)\Big]-2T\left[-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right. \\
&\quad \left.\left.+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+T^2\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]\right\} \\
&\quad +a\left[\Phi\left(\frac{T_L-\mu}{\sigma}\right)+1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+\left(a-b\frac{T_L}{T-T_L}\right)\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right] \\
&\quad +\frac{b}{T-T_L}\left[\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)\right]+ \\
&\quad \left(a+b\frac{T_U}{T_U-T}\right)\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]-\frac{b}{T_U-T}\left[\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\right. \\
&\quad \left.\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)\right] \qquad (6)
\end{aligned}
$$

The detail derivation of Equation (6) is shown in Appendix A. It is difficult to show a closed-form solution for Equation (6) because the total losses to society per item, $C_{T_1}$, includes $\phi$ and $\Phi$. For the given parameters $k_1$, $k_2$, $T$, $T_L$, $T_U$, $\sigma$, $a$, $b$, $D_L$, and $D_U$, the optimum process mean $\mu^*$ for Equation (6) can been obtained by using the direct search method. One can set $\mu$ values from $T_L - 2$, through $T_U + 2$, in step 0.0001 for example, in order to search the minimum $C_{T_1}$.

*With Manufacturing Cost and Linear Quality Loss Function*

Using the same assumption as 5.1, here the objective is to minimize the following total losses to society per item consisting of the producer's manufacturing cost and the customer's linear quality loss,

$$
\begin{aligned}
&C_{T_2} \\
&= D_U\left[1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+D_L\Phi\left(\frac{T_L-\mu}{\sigma}\right)+k_1T\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]
\end{aligned}
$$

$$-k_1\left[-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)$$

$$-k_2T\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+k_2\left[-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]$$

$$+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\Bigg]$$

$$+a\left[\Phi\left(\frac{T_L-\mu}{\sigma}\right)+1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+\left(a-b\frac{T_L}{T-T_L}\right)\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]$$

$$+\frac{b}{T-T_L}\left[\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T_L-\mu}{\sigma s}\right)\right]+$$

$$\left(a+b\frac{T_U}{T_U-T}\right)\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]-\frac{b}{T_U-T}\left[\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\right.$$

$$\left.\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)\right]$$

The detail derivation of Equation (7) is shown in Appendix B. The optimum process mean $\mu^*$ for Equation (7) can been obtained by using the same manner as in the previous subsection.

## NUMERICAL EXAMPLE

A production line which is manufacturing ball bearings in a factory, the grinding process creases the bore of the inner ring. The specifications of the bore diameter are 8 and 12 mm. Suppose that in this process, when the bore diameter is greater than 12 mm, the inner rings would be waste, and when the bore diameter is less than 8 mm, the inner rings could be recycled. The loss per waste inner ring is \$25; the loss per recycled inner ring is \$18. The inner ring bore diameter follows a normal distribution and the standard deviation of the process is estimated as $\sigma = 0.5$. The target value of inner ring $T = 9.5$ mm.

The constant manufacturing cost per item $a = 5$ and the variable manufacturing cost coefficient per item $b = 9$. The manufacturing cost is negatively and linearly proportional to deviation from the target value when the quality characteristic is within specifications and constant manufacturing cost happens when the quality characteristic exceeds specifications. The objective is to find an optimal process mean that it will be at the optimum level possible, while the same time minimizing the manufacturing cost and quality loss.

*With Manufacturing Cost and Quadratic Quality Loss Function*

The quality loss coefficient $k_1$ to represent the left direction of deviation from the target value $T$ is $k_1 = \frac{18}{(1.5)^2}$ The quality loss coefficient $k_2$ for representing the right direction

of deviation from the target value $T$ is k2 = $\frac{25}{(2.5)^2}$ = 4. By solving Equation (6), the optimum process mean for the nominal-is-best type normal quality characteristic is $\mu^*$ = 9.5972 with the optimum total losses to society per item $C^*_{T_1}$ = 13.5643.

*With Manufacturing Cost and Linear Quality Loss Function*

The quality loss coefficient $k_1$ for representing the left direction of deviation from the target value $T$ is k1 = $\frac{18}{1.5}$ = 12. The quality loss coefficient $k_2$ for representing the right direction of deviation from the target value $T$ is k2 = $\frac{25}{2.5}$ = 10. By solving Equation (7), the optimum process mean for the nominal-is-best type normal quality characteristic is m* = 9.9844 with the optimum total losses to society per item $C^*_{T_2}$ = 14.3005.

From the above numerical example, we have the following results:

1. The large $k_1$ and $k_2$ values have large total losses to society per item. That is because the large quality loss coefficients $k_1$ and $k_2$ values usually show the large quality cost within specifications. Hence, the larger $k_1$ and $k_2$ values, the larger total losses to society per item will be.
2. The linear quality loss function of modified model has larger total losses to society per item than that of quadratic quality loss of modified model. The former shows larger quality loss within specifications than that of the latter. We need to consider which quality loss function should be adopted for measuring the product quality within specifications.
3. The large total losses to society per item represent the large opportunity cost to the society. The optimum process mean setting is an approach for improving product. For the long-term, we need to improve the production process to obtain the optimum output value of product. One needs to address Taguchi's (1986) parameter design method of off-line quality control, adopt statistical process control method and apply the advanced manufacturing technology to reduce the total losses to society.

## CONCLUSIONS

The mathematical models for determining the optimum process mean of modified Wen and Mergen's (1999) model have been proposed in this article. The process target value may be different from the design target value because the optimum value setting of the former is based on minimizing the total loss to society per item consisting of the producer's manufacturing cost and the customer's quality loss. Further direction of study should address the modified model with manufacturing cost and quality loss for a one-sided specification limit product.

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## Appendix A: The Detail Derivation of Equation (6)

$$
\begin{aligned}
&C_{T_1} \\
&= D_L \int_{-\infty}^{T_L} f(x)dx + \int_{T_L}^{T} k_1(T-x)^2 f(x)dx + \int_{T}^{T_U} k_2(x-T)^2 f(x)dx + \\
&D_U \int_{T_U}^{\infty} f(x)dx + a\int_{-\infty}^{T_L} f(x)dx + \int_{T_L}^{T}\left[a+b\frac{x-T_L}{T-T_L}\right]f(x)dx + \\
&\int_{T}^{T_U}\left[a+b\frac{T_U-x}{T_U-T}\right]f(x)dx + a\int_{T_U}^{\infty} f(x)dx
\end{aligned} \tag{A1}
$$

We have

$$
\begin{aligned}
&\int_{T_L}^{T} k_1(T-x)^2 f(x)dx \\
&= k_1\left\{\left[\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T-\mu}{\sigma}\right) - \sigma(T+\mu)\phi\left(\frac{T-\mu}{\sigma}\right) - \left(\sigma^2+\mu^2\right)\Phi\left(\frac{T_L-\mu}{\sigma}\right) + \right.\right. \\
&\sigma(T_L+\mu)\phi\left(\frac{T_L-\mu}{\sigma}\right)\Big] - 2T\left[-\sigma\phi\left(\frac{T-\mu}{\sigma}\right) + \mu\Phi\left(\frac{T-\mu}{\sigma}\right) + \sigma\phi\left(\frac{T_L-\mu}{\sigma}\right) - \right. \\
&\left.\left.\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right] + T^2\left[\Phi\left(\frac{T-\mu}{\sigma}\right) - \Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]\right\}
\end{aligned} \tag{A2}
$$

$$
\begin{aligned}
&\int_{T}^{T_U} k_2(x-T)^2 f(x)dx \\
&= k_2\left\{\left[\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T_U-\mu}{\sigma}\right) - \sigma(T_U+\mu)\phi\left(\frac{T_U-\mu}{\sigma}\right) - \left(\sigma^2+\mu^2\right)\Phi\left(\frac{T-\mu}{\sigma}\right) + \right.\right. \\
&\sigma(T+\mu)\phi\left(\frac{T-\mu}{\sigma}\right)\Big] - 2T\left[-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right) + \mu\Phi\left(\frac{T_U-\mu}{\sigma}\right) + \sigma\phi\left(\frac{T-\mu}{\sigma}\right) - \right. \\
&\left.\left.\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\right] + T^2\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right) - \Phi\left(\frac{T-\mu}{\sigma}\right)\right]\right\}
\end{aligned} \tag{A3}
$$

$$\int_{T_L}^{T}\left[a+b\frac{x-T_L}{T-T_L}\right]f(x)dx$$

$$= \left(a-b\frac{T_L}{T-T_L}\right)\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+\frac{b}{T-T_L}\left[\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\right.$$

$$\left.\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)\right] \qquad \text{(A4)}$$

$$\int_{T}^{T_U}\left[a+b\frac{T_U-x}{T_U-T}\right]f(x)dx$$

$$= \left(a+b\frac{T_U}{T_U-T}\right)\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]-\frac{b}{T_U-T}\left[\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\right.$$

$$\left.\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)\right] \qquad \text{(A5)}$$

Substituting Equations (A2)-(A5) into (A1), we have

$$C_{T_1}$$

$$= D_U\left[1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+D_L\Phi\left(\frac{T_L-\mu}{\sigma}\right)+k_1\left\{\left[\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T-\mu}{\sigma}\right)-\sigma(T+\mu)\phi\left(\frac{T-\mu}{\sigma}\right)\right.\right.$$

$$-\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T_L-\mu}{\sigma}\right)+\sigma(T_L+\mu)\phi\left(\frac{T_L-\mu}{\sigma}\right)\Big]-2T\left[-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\right.$$

$$\left.+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]+T^2\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]\Big\}$$

$$k_2\left\{\left[\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\sigma(T_U+\mu)\phi\left(\frac{T_U-\mu}{\sigma}\right)\right.\right.$$

$$-\left(\sigma^2+\mu^2\right)\Phi\left(\frac{T-\mu}{\sigma}\right)+\sigma(T+\mu)\phi\left(\frac{T-\mu}{\sigma}\right)\Big]-2T\left[-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right.$$

$$\left.+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+T^2\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]\Big\}$$

$$+a\left[\Phi\left(\frac{T_L-\mu}{\sigma}\right)+1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+\left(a-b\frac{T_L}{T-T_L}\right)\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]$$

$$+\frac{b}{T-T_L}\left[\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)\right]+$$

$$\left(a+b\frac{T_U}{T_U-T}\right)\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]-\frac{b}{T_U-T}\left[\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\right.$$
$$\left.\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)\right] \quad \text{(A6)}$$

## APPENDIX B: THE DETAIL DERIVATION OF EQUATION (7)

$$C_{T_2}$$
$$= D_L\int_{-\infty}^{T_L} f(x)dx+\int_{T_L}^{T} k_1(T-x)f(x)dx+\int_{T}^{T_U} k_2(x-T)f(x)dx+D_U\int_{T_U}^{\infty} f(x)dx+$$
$$a\int_{-\infty}^{T_L} f(x)dx+\int_{T_L}^{T}\left[a+b\frac{x-T_L}{T-T_L}\right]f(x)dx+\int_{T}^{T_U}\left[a+b\frac{T_U-x}{T_U-T}\right]f(x)dx+a\int_{T_U}^{\infty} f(x)dx \quad \text{(B1)}$$

We have

$$\int_{T_L}^{T} k_1(T-x)f(x)dx$$
$$= k_1T\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]--k_1\left[-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T-\mu}{\mu}\right)\right]+$$
$$\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)\Bigg] \quad \text{(B2)}$$

$$\int_{T}^{T_U} k_2(x-T)f(x)dx$$
$$= -k_2T\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+k_2\left[-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]$$
$$+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\Bigg] \quad \text{(B3)}$$

$$\int_{T_L}^{T}\left[a+b\frac{x-T_L}{T-T_L}\right]f(x)dx$$
$$= \left(a-b\frac{T_L}{T-T_L}\right)\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]+\frac{b}{T-T_L}\left[\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\right.$$

$$\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)\Bigg] \tag{B4}$$

$$\int_{T}^{T_U}\left[a+b\frac{T_U-x}{T_U-T}\right]f(x)dx$$

$$= \left(a+b\frac{T_U}{T_U-T}\right)\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]-\frac{b}{T_U-T}\left[\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\right.$$

$$\left.\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)\right] \tag{B5}$$

Substituting Equations (B2)-(B5) into (B1), we have

$$C_{T_2}$$

$$= D_U\left[1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+D_L\Phi\left(\frac{T_L-\mu}{\sigma}\right)+k_1T\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]$$

$$-k_1\left[-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)\Bigg]$$

$$-k_2T\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]+k_2\left[-\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\mu\Phi\left(\frac{T_U+\mu}{\sigma}\right)\right]$$

$$+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)\Bigg]$$

$$+a\left[\Phi\left(\frac{T_L-\mu}{\sigma}\right)+1-\Phi\left(\frac{T_U-\mu}{\sigma}\right)\right]+\left(a-b\frac{T_L}{T-T_L}\right)\left[\Phi\left(\frac{T-\mu}{\sigma}\right)-\Phi\left(\frac{T_L-\mu}{\sigma}\right)\right]$$

$$+\frac{b}{T-T_L}\left[\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T_L-\mu}{\sigma}\right)-\sigma\phi\left(\frac{T-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T_L-\mu}{\sigma}\right)\right]+$$

$$\left(a+b\frac{T_U}{T_U-T}\right)\left[\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\Phi\left(\frac{T-\mu}{\sigma}\right)\right]-\frac{b}{T_U-T}\left[\mu\Phi\left(\frac{T_U-\mu}{\sigma}\right)-\mu\Phi\left(\frac{T-\mu}{\sigma}\right)-\right.$$

$$\left.\sigma\phi\left(\frac{T_U-\mu}{\sigma}\right)+\sigma\phi\left(\frac{T-\mu}{\sigma}\right)\right] \tag{B6}$$